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Partial stabilization-based guidance

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ABSTRACT

A novel nonlinear missile guidance law against maneuvering targets is designed based on the principles of partial stability. It is demonstrated that in a real approach which is adopted with actual situations, each state of the guidance system must have a special behavior and asymptotic stability or exponential stability of all states is not realistic. Thus, a new guidance law is developed based on the partial stability theorem in such a way that the behaviors of states in the closed-loop system are in conformity with a real guidance scenario that leads to collision. The performance of the proposed guidance law in terms of interception time and control effort is compared with the sliding mode guidance law by means of numerical simulations.

1. Introduction

Recently, nonlinear control theories have been used in the design of robust guidance laws. Methods such as Lyapunov-based nonlinear guidance laws [1–3], first-order sliding mode guidance laws [4–8] and nonlinear H_∞ guidance laws [9,10] have been obtained from Lyapunov theorems on asymptotic stability or exponential stability of all states. In this paper, it is shown that, in practice, such a behavior is not realistic for all the states of a guidance system.

It is worth noting that although the missile guidance problem can be treated in the class of uncertain nonlinear systems, it has a characteristic property: the missile should intercept a highly maneuvering target within a finite interception time, sometimes only a few seconds. In this paper, a guidance law will be designed which guarantees the mentioned characteristic. The present work may be classified in the category of robust guidance laws; however, in comparison to other robust guidance laws, it follows a completely different approach towards the missile guidance problem. It is shown that, in a practical approach to the guidance problem, each state has a specific behavior and there is no need for asymptotic convergence of all the states. This is in contrast to conventional methods in nonlinear control theory which try to force all the states to asymptotically converge to the origin (equilibrium point). Indeed, this paper presents a new approach in nonlinear guidance law design. The approach is advantageous from a practical view-point since it forces the states to behave as they need in practice and leads to classification of the state variables of the guidance system dynamic with respect to their required stability. The proposed guidance law is based on the principle of *partial stability*, which is stability with respect to some of the state variables.

For many engineering problems like inertial navigation systems, spacecraft stabilization via gimbaled gyroscopes or flywheels, electromagnetic, adaptive stabilization, etc. [11–14], the provision of *partial stability* is necessary. In the mentioned applications, while the plant may be unstable in the standard sense, it is partially and not totally asymptotically stable. The effectiveness of the proposed guidance law in intercepting a highly maneuvering target with zero-miss-distance and finite interception time is demonstrated analytically and through computer simulation.

2. Partial stability analysis

Partial stability is defined as the stability of a dynamical system with respect to only some of the state variables. This approach is essential for many engineering fields [14]. Consider a nonlinear dynamical system:

$$\dot{x} = f(x), \qquad x(t_0) = x_0 \tag{1}$$

where $x \in R^n$ is the state vector. Let vectors x_1 and x_2 denote the partitions of the state vector, respectively. Since $x = (x_1^T, x_2^T)^T$, $x_1 \in R^{n_1}$, $x_2 \in R^{n_2}$, and $n_1 + n_2 = n$, the nonlinear system (1) can be divided into two parts:

$$\begin{aligned} \dot{x}_1(t) &= F_1(x_1(t), x_2(t)), \qquad x_1(t_0) = x_{10} \\ \dot{x}_2(t) &= F_2(x_1(t), x_2(t)), \qquad x_2(t_0) = x_{20} \end{aligned} \tag{2}$$

where $x_1 \in D \subseteq R^{n_1}$, D is an open set including the origin, $x_2 \in R^{n_2}$ and $F_1 : D \times R^{n_2} \to R^{n_1}$ is such that for every $x_2 \in R^{n_2}$, $F_1(0, x_2) = 0$

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and $F_1(., x_2)$ is locally Lipschitz in x_1. Also, $F_2 : D \times R^{n_2} \to R^{n_2}$ is such that for every $x_1 \in D$, $F_2(x_1, .)$ is locally Lipschitz in x_2 and $I_{x_0} = [0, \tau_{x_0}), 0 < \tau_{x_0} \leq \infty$ is the maximal solution existence interval $(x_1(t), x_2(t))$ of (2) $\forall t \in I_{x_0}$. Under these conditions, the existence and uniqueness of the solution is ensured. Now, the stability of the dynamical system (2) with respect to x_1 can be defined as follows [14].

Definition 1. (i) The nonlinear system (2) is Lyapunov stable with respect to x_1 if for every $\varepsilon > 0$ and $x_{20} \in R^{n_2}$, there exists $\delta(\varepsilon, x_{20}) > 0$ such that $\|x_{10}\| < \delta$ implies $\|x_1(t)\| < \varepsilon$ for all $t \geq 0$.

(ii) The nonlinear system (2) is asymptotically stable with respect to x_1, if it is Lyapunov stable with respect to x_1 and for every $x_{20} \in R^{n_2}$, there exists $\delta = \delta(x_{20}) > 0$ such that $\|x_{10}\| < \delta$ implies $\lim_{t\to\infty} x_1(t) = 0$.

In order to analyze partial stability, the following theorem and its corollary are taken from [14].

Theorem 1. *Nonlinear dynamical system (2) is asymptotically stable with respect to x_1 if there exist a continuously differentiable function $V : D \times R^{n_2} \to R$ and class K functions $\alpha(.)$ and $\gamma(.)$, such that*

$$V(0, x_2) = 0, \quad x_2 \in R^{n_2} \tag{3}$$

$$\alpha(\|x_1\|) \leq V(x_1, x_2), \quad (x_1, x_2) \in D \times R^{n_2} \tag{4}$$

$$\frac{\partial V(x_1, x_2)}{\partial x_1} F_1(x_1, x_2) + \frac{\partial V(x_1, x_2)}{\partial x_2} F_2(x_1, x_2) \leq -\gamma(\|x_1\|), \quad (x_1, x_2) \in D \times R^{n_2}. \tag{5}$$

Proof. See [14]. □

Corollary 1. *Consider the nonlinear dynamical system (2). If there exist a positive definite, continuously differentiable function $V : D \to R$ and a class K function $\gamma(.)$, such that*

$$\frac{\partial V(x_1)}{\partial x_1} F_1(x_1, x_2) \leq -\gamma(\|x_1\|) \quad (x_1, x_2) \in D \times R^{n_2} \tag{6}$$

then the equilibrium point of the nonlinear dynamical system (2) is asymptotically stable with respect to x_1.

Proof. See [14]. □

3. Application of partial stability to the missile guidance problem

3.1. Problem formulation

In this section, a two dimensional air to air engagement is considered. The vehicles are modeled as point masses and the relative kinematics equation of motion between the missile and the target is derived in the planner line of sight (LOS) coordinate system. The center of the planner LOS coordinate system is located on the missile. The first axis (X_L) is along the LOS vector (i.e. the vector connecting the missile to the target at each moment and its direction is toward the target). The second axis (Y_L) is perpendicular to the first one. The components of the missile and target speed vectors ($\vec{v}_M$ and $\vec{v}_T$) in the LOS coordinate system are shown in Fig. 1.

The representation of the LOS vector in the LOS coordinate system is as follows:

$$\vec{r} = \begin{bmatrix} r \\ 0 \end{bmatrix} \tag{7}$$

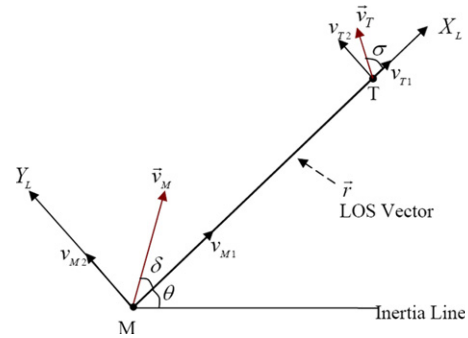

Fig. 1. LOS coordinate system.

where r is the relative distance between the missile and the target. Also, the LOS vector makes an angle of θ with respect to the inertia reference line. The LOS coordinate system rotates with respect to the inertia reference line and $\vec{\omega}_{IL}$ – defined below – denotes the rotation rate of the LOS-frame with respect to the inertia-frame. The only nonzero element of this vector is perpendicular to the plane of the LOS coordinate system:

$$\vec{\omega}_{IL} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix}. \tag{8}$$

The elements of the relative speed vector and the relative acceleration vector in the LOS coordinate are

$${}^L\vec{v}_r = {}^L(\vec{v}_T - \vec{v}_M) = \begin{bmatrix} v_{T1} - v_{M1} \\ v_{T2} - v_{M2} \end{bmatrix} = \begin{bmatrix} v_{r1} \\ v_{r2} \end{bmatrix} \tag{9}$$

$${}^L\vec{a}_r = {}^L(\vec{a}_T - \vec{a}_M) = \begin{bmatrix} a_{T1} - a_{M1} \\ a_{T2} - a_{M2} \end{bmatrix} = \begin{bmatrix} a_{r1} \\ a_{r2} \end{bmatrix}. \tag{10}$$

Now, by differentiating with respect to time and use of the Coriolis Theorem [15] the relative equations of motion in the LOS coordinate can be obtained as follows:

$${}^L\vec{v}_r = {}^L(D_I\vec{r}) = {}^L(D_L\vec{r}) + {}^L\vec{\omega}_{IL} \times {}^L\vec{r} = \begin{bmatrix} \dot{r} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} \times \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{r} \\ r\dot{\theta} \\ 0 \end{bmatrix} \tag{11}$$

where ${}^L\vec{v}_r$ is the relative speed vector represented in the LOS coordinate. D_I is the differential operator with respect to the inertia coordinate and D_L is the differential operator with respect to the LOS coordinate.

Note. Since the inner product in the Coriolis Theorem is used, the LOS vector and its derivation are shown with three elements. Of course, the third element is always equal to zero.

Therefore, the elements of the relative speed vector in the LOS coordinate are

$$\begin{bmatrix} \dot{r} \\ r\dot{\theta} \end{bmatrix} = \begin{bmatrix} v_{T1} - v_{M1} \\ v_{T2} - v_{M2} \end{bmatrix} = \begin{bmatrix} v_{r1} \\ v_{r2} \end{bmatrix} \tag{12}$$

where $\dot{r}$ and $r\dot{\theta}$ are the radial and tangential components of the relative speed vector, respectively. Now, by differentiating the relative speed vector, the acceleration elements in the LOS coordinate can be obtained:

$${}^L\vec{a}_r = {}^L(D_I\vec{v}_r) = {}^L(D_L\vec{v}_r) + {}^L\vec{\omega}_{IL} \times {}^L\vec{v}_r = \begin{bmatrix} \ddot{r} \\ r\ddot{\theta} + \dot{r}\dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} \times \begin{bmatrix} \dot{r} \\ r\dot{\theta} \\ 0 \end{bmatrix} = \begin{bmatrix} \ddot{r} - r\dot{\theta}^2 \\ r\ddot{\theta} + 2\dot{r}\dot{\theta} \\ 0 \end{bmatrix}. \tag{13}$$

As a result, the elements of relative speed in the LOS coordinate are

$$\begin{bmatrix} \ddot{r} - r\dot{\theta}^2 \\ r\ddot{\theta} + 2\dot{r}\dot{\theta} \end{bmatrix} = \begin{bmatrix} a_{T1} - a_{M1} \\ a_{T2} - a_{M2} \end{bmatrix} = \begin{bmatrix} a_{r1} \\ a_{r2} \end{bmatrix}. \tag{14}$$

The above equations are defined as engagement equations. By considering $[V_r \quad V_\theta] = [\dot{r} \quad r\dot{\theta}]$, Eq. (14) can be rewritten in the following way:

$$\frac{d}{dt}\begin{bmatrix} r \\ V_r \\ V_\theta \end{bmatrix} = \begin{bmatrix} V_r \\ V_\theta^2/r \\ -V_r V_\theta/r \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} w \tag{15}$$

where $x = [r, V_r, V_\theta]^T$ is the state vector, $w = (w_1, w_2)^T = (a_{T1}, a_{T2})^T$ and $u = (u_1, u_2)^T = (a_{M1}, a_{M2})^T$ are the acceleration vectors of the target and missile, respectively. Here, the acceleration of the target is assumed as an external bounded disturbance and only this bound is required in the design of the guidance law and the accurate measurement of target acceleration during the maneuver is not necessary. The reason for choosing $r\dot{\theta}$ instead of $\dot{\theta}$ as the third state variable is that it avoids the appearance of the term $1/r$ in the control and disturbance coefficient matrix. Thus, such a selection would make these matrices constant.

Remark 1. The initial conditions of the terminal phase are usually in such a way that $r_0 > 0$ and $V_{r0} < 0$ (as assumed in the present work). This means that the target is in front of the missile and the missile is approaching it.

3.2. Desirable behaviors for each state

For the interception, it is sufficient that $r(t)$ becomes zero at an instance ($r(t_f) = 0$ where t_f is the interception time) and there is no need for $r(t)$ to asymptotically converge to zero. In other words, asymptotic convergence is not an ideal behavior for $r(t)$. It should be noted that asymptotic convergence behavior means that the missile initially approaches the target very fast; however, near the target, the relative distance reduces so slowly that the missile touches the target in an infinite time. It is evident that such a behavior is not a desirable behavior in missile guidance which is intended to hit the target with a non-zero speed in a finite time. For this purpose, it is sufficient for the relative radial speed between the missile and the target to satisfy the following proposition.

Proposition 1. *In order to intercept the target within a finite time interval, it is sufficient to have*

$$\exists t_1 \in [t_0 t_f) \quad s.t.\ V_r(t) \le -\zeta < 0 \quad \forall t \in [t_1 t_f]. \tag{16}$$

Proof. $r(t)$ is positive and continuous. Consequently, to reach a zero value, i.e. zero-miss-distance, within a finite time, it should decrease from its value at $t_1(<t_f)$ down to a zero value at t_f. Inequality (16) implies $r(t) - r(t_1) \le -\zeta(t - t_1)$, which shows that $r(t)$ reduces within the time interval $[t_1 \quad t]$. Since it is desirable to have $r(t_f) = 0$, thus

$$t_f \le \frac{r(t_1) + \zeta t_1}{\zeta}. \tag{17}$$

Hence, the expectation is that V_r satisfies Proposition 1. In other words, the convergence and stability of this state are not under consideration. □

Remark 2. A common approach in relevant papers is to regulate V_r to a negative constant c. Although this approach guarantees the interception of the non-maneuvering target within a finite time, it is not efficient for intercepting a highly maneuvering target in an acceptable interception time. In such a case, to improve the performance, c should vary with time; however, to determine it, the maneuvering model of the target should be known for the missile, which is not always practically possible.

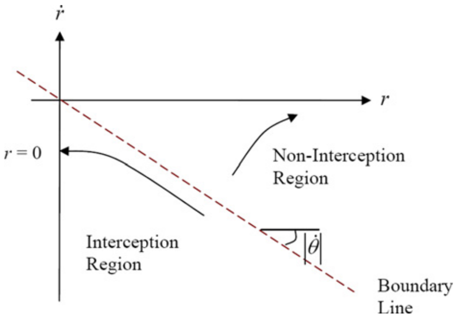

Fig. 2. Interception and non-interception regions.

For the third state variable, V_θ, asymptotic convergence behavior to zero is desirable. This results in equality of v_{M2} and v_{T2}, that is equivalent to stating that the LOS vectors remain parallel with each other (stopping in LOS rotation). The following discussion will clarify the reason for the acceptability of such a behavior [16].

Consider the first equation of engagement equations (14):

$$\ddot{r} = r\dot{\theta}^2 + (a_{T1} - a_{M1}). \tag{18}$$

Assume $\dot{\theta}$ is temporarily fixed and the resulting radial acceleration components are negligible. Then the solution of this differential equation is as follows:

$$r = c_1 e^{\alpha t} + c_2 e^{-\alpha t} \tag{19}$$

where $c_1 = 0.5(r_0 + V_{r0}/\alpha)$, $\alpha = |\dot{\theta}|$ and $r_0,\ V_{r0}$ are the initial conditions of engagement. Usually, these initial conditions are such that $r_0 > 0$ and $V_{r0} < 0$ (refer to Remark 1). This means that the target is in front of the missile and the missile is approaching it. Based on these initial values, three cases might occur (in the third quarter of the $r - V_r$ plane).

- If $c_1 < 0$ or $r_0 < -V_{r0}/\alpha$, then the first term in expression (19) is negative which results in reduction of the relative distance (r) with time. Therefore, the missile will intercept the target. So, this area is called the "interception region".
- If $c_1 > 0$ or $r_0 > -V_{r0}/\alpha$, then the relative distance increases with time and the missile does not intercept the target. This area is called the "non-interception region".
- If $c_1 = 0$ or $r_0 = -V_{r0}/\alpha$, then there is a linear relation between r and V_r at all moments. This line is the boundary between the interception and non-interception regions.

These regions are shown in Fig. 2. As observed, it is obvious that a smaller $|\dot{\theta}|$ will result in a larger interception region. Thus, it is desirable that $\dot{\theta}$ converges to zero (quickly) and stays on it. As a result, asymptotic stability behavior is desirable for convergence of $\dot{\theta}$. Also, the appropriate behavior for V_θ is the same. This is because stopping in LOS rotation leads to a zero value for V_θ.

Therefore, the state vector may be separated into $x_1 = V_\theta$ and $x_2 = [r \quad V_r]$ where asymptotic stability behavior only for x_1 is desirable. By modeling the system (14) in $x_1 - x_2$ coordinates, the following can be obtained:

$$\begin{aligned} \dot{x}_1 &= \frac{-V_r V_\theta}{r} - u_2 + w_2 \\ \dot{x}_2 &= \begin{bmatrix} V_r \\ \dfrac{V_\theta^2}{r} - u_1 + w_1 \end{bmatrix}. \end{aligned} \tag{20}$$

It is evident that only the second component of the input vector appears in the $\dot{x}_1$-equation. Therefore, the asymptotic convergence of V_θ may be achieved by u_2. In the meantime, the appropriate behavior of x_2 could be obtained by u_1.

3.3. Guidance law design

First, consider Eq. (15) where $w = 0$. This is equivalent to a non-maneuvering target. Taking the Lyapunov function to be $V(x_1) = 0.5x_1^2 = 0.5V_\theta^2$, the time derivative of $V(x_1)$ in the line of the system's trajectory is

$$\dot{V}(x_1) = V_\theta \left(\frac{-V_r V_\theta}{r} - u_2 \right). \tag{21}$$

Take

$$u_2 = \frac{-V_r V_\theta}{r} + NV_\theta. \tag{22}$$

Thus, the derivative of $V(x_1)$ will satisfy the condition $\dot{V}(x_1) \le -\gamma(\|x_1\|)$ where $\gamma(\|x_1\|) = N V_\theta^2$. Therefore, according to Corollary 1, asymptotic convergence towards zero is achieved for x_1. Also, by choosing

$$u_1 = \frac{V_\theta^2}{r} - \sigma V_r \quad \sigma > 0, \tag{23}$$

one has $\dot{V}_r = \sigma V_r$, and consequently

$$V_r(t) = V_{r0} e^{\sigma t} \tag{24}$$

where $V_{r0} < 0$ (according to Remark 1). As a result, $V_r(t) \le V_{r0} < 0$. Therefore, Proposition 1 is satisfied.

Remark 3. Clearly, higher values of σ cause a shorter interception time; however, its adjustment should be made with respect to the physical limitations. It is worth noting that σ adjustment can also be made in an adaptive manner and it may decrease as r decreases. For example, it can decrease linearly as follows:

$$\sigma(t) = \left[\frac{r(t)}{r_0} \right] \sigma_0 + \left[1 - \frac{r(t)}{r_0} \right] \sigma_f \tag{25}$$

where σ_0 and σ_f denote the initial and final values of σ, respectively.

Now, assume $w \neq 0$. In this case, the target may have any arbitrary maneuver with a bounded acceleration. Additional control components, v_1 and v_2, may be designed in such a way as to make the guidance law robust with respect to w. By taking

$$\begin{aligned} u_{2_{new}}(x) &= u_2(x) + v_2(x) \\ &= -\frac{V_r V_\theta}{r} + NV_\theta + v_2(x) \end{aligned} \tag{26}$$

one has

$$\dot{V}(x_1) = -NV_\theta^2 - V_\theta(v_2 - w_2) \tag{27}$$

where the last term, i.e., $-V_\theta(v_2 - w_2)$, is the effect of the control component (v_2) and disturbance term (w_2). Assume $|w_2| \le \eta_2$; therefore,

$$\begin{aligned} -V_\theta(v_2 - w_2) &\le -V_\theta v_2 + |V_\theta||w_2| \\ &\le -V_\theta v_2 + \eta_2 |V_\theta|. \end{aligned} \tag{28}$$

By choosing $v_2 = \eta_2 \,\mathrm{sgn}(V_\theta)$, one has

$$-V_\theta(v_2 - w_2) \le -\eta_2|V_\theta| + \eta_2|V_\theta| = 0. \tag{29}$$

Thus, $\dot{V}(x_1) \le -\gamma(\|x_1\|)$ when w_2 is present. Now, the additional term, v_1, could be designed in such a way that the control law, $u_{1_{new}}(x) = u_1(x) + v_1(x)$, guarantees the specified behavior for x_2 in the presence of w_1.

$$u_{1_{new}}(x) = \frac{V_\theta^2}{r} - \sigma V_r + v_1. \tag{30}$$

Assume $|w_1| \le \eta_1$, and take $v_1 = -\eta_1 \,\mathrm{sgn}(V_r)$. Since V_r is supposed to be negative, thus $\mathrm{sgn}(V_r) = -1$ and $v_1 = \eta_1$. By inserting $u_{1_{new}}$ in the dynamic equation (15), one has:

$$\dot{V}_r = \sigma V_r - \eta_1 + w_1. \tag{31}$$

Thus,

$$\begin{aligned} V_r(t) &= V_{r0} e^{\sigma t} + \int_0^t (-\eta_1 + w_1) e^{\sigma(t-\tau)} d\tau \\ &\le V_{r0} e^{\sigma t} - \int_0^t \eta_1 e^{\sigma(t-\tau)} d\tau + \int_0^t \eta_1 e^{\sigma(t-\tau)} d\tau \\ &\le V_{r0} e^{\sigma t}. \end{aligned} \tag{32}$$

Choosing $-\zeta = V_{r0} < 0$ results in $V_r(t) \le -\zeta < 0$ for $t \in [0\ t_f]$, for which Proposition 1 is satisfied.

Remark 4. Since discontinuous controllers suffer from chattering, one way to alleviate this problem is to consider an approximation of the signum function by a saturation function with a high slope ($\frac{1}{\varepsilon}$).

Consequently, the guidance law (33) guarantees interception of the maneuvering target within a finite interception and zero-miss distance:

$$\begin{cases} u_{1_{new}}(x) = \dfrac{V_\theta^2}{r} - \sigma V_r + \eta_1 \quad \sigma, \eta_1 > 0 \\ u_{2_{new}}(x) = -\dfrac{V_\theta V_r}{r} + NV_\theta + \eta_2 sat\left(\dfrac{V_\theta}{\varepsilon}\right) \quad N, \varepsilon, \eta_2 > 0. \end{cases} \tag{33}$$

3.4. Computer simulations

Numerical simulations are performed to illustrate the effectiveness of the designed nonlinear guidance law.

The initial conditions of engagement are specified as

$$\begin{aligned} & r(0) = 5000 \text{ m}, \\ & V_r(0) = -300 \text{ m/s}, \\ & V_\theta(0) = 100/\text{s} \end{aligned} \tag{34}$$

and the designed guidance law (33) is compared with the sliding mode guidance law presented by the following guidance command [8]:

$$\begin{aligned} u_1 &= \frac{V_\theta^2}{r} + \eta_1 \,\mathrm{sat}\left(\frac{V_r - c}{\varepsilon}\right) \\ u_2 &= -(N+1)\frac{V_r V_\theta}{r} + \eta_2 \,\mathrm{sat}\left(\frac{V_\theta}{\varepsilon}\right). \end{aligned} \tag{35}$$

The sliding mode guidance law has been compared with some existing guidance laws [8]. This guidance law has obtained better performance in terms of interception time and control effort compared to other existing guidance laws. In this paper, it is shown that our proposed guidance law achieves better performance compared to the sliding mode guidance law. In both cases, a highly maneuvering target with the following acceleration vector is considered:

$$a_T(t) = \begin{bmatrix} 200 \sin \dfrac{\pi}{6} t \\ 200 \sin \dfrac{\pi}{4} t \end{bmatrix}. \tag{36}$$

The initial conditions in the inertia-frame of the missile and the target for nonlinear simulation are given by the following.

- For the missile: $x_M(0) = y_M(0) = 0$ m.
- For the target: $x_T(0) = 5000 \cos\theta_0$ m, $y_T(0) = 5000 \sin\theta_0$ m, $v_{Tx}(0) = -200$ m/s, $v_{Ty}(0) = 0$ m/s.

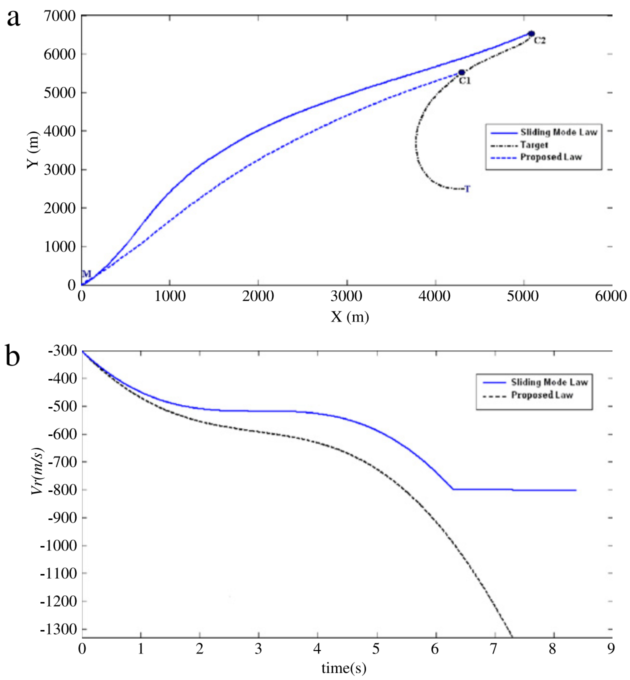

Fig. 3. (a) Trajectories of the missile and target. (b) Relative radial speed (V_r).

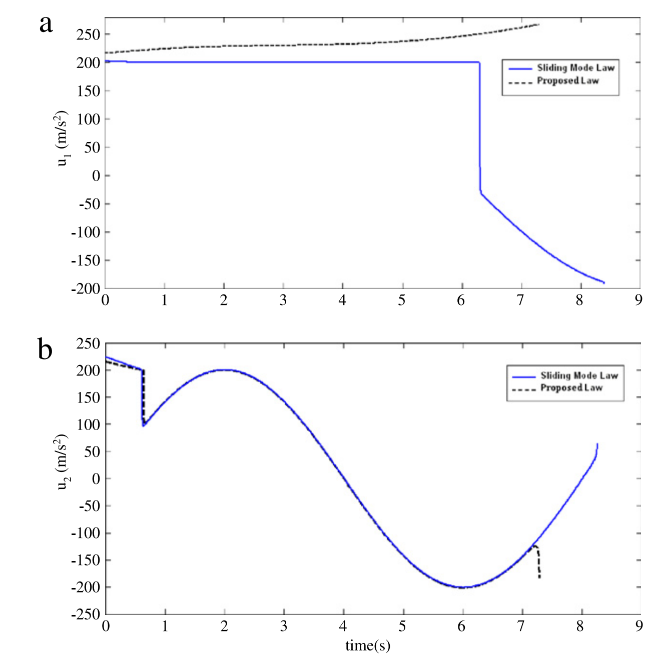

Fig. 4. (a) Radial guidance command (u_1). (b) Tangential guidance command (u_2).

θ_0 is set to be $\pi/6$. The initial values of the missile speed components can be obtained based on the initial relative speed components (34) and the initial values of target speeds.

The constant N is chosen as 4, while η_1 and η_2 are set to be 200. Also, $\varepsilon = 1$, $\beta = 0.1$, $\sigma = 0.05$ and $c = -800$ are selected. Fig. 3(a) shows the trajectories of the missile and the target and Fig. 3(b) depicts the relative radial speed (V_r). Fig. 4(a) and (b) reflect the guidance commands.

Fig. 3(a) illustrates the collision points, C1 and C2, for the proposed guidance law and the sliding mode guidance law, respectively. The interception time is 7.26 s for the proposed guidance law and 8.38 s for the sliding mode guidance law. Fig. 4(a) demonstrates that the amount of change of radial control effort for the proposed law is less than for the sliding mode guidance law. Also Fig. 4(b) shows that the proposed guidance law requires less tangential control effort than the sliding mode guidance law.

4. Conclusion

In this paper, a new approach to the missile guidance problem was presented. It became clear that in a successful missile guidance scenario, which leads to target interception, the desirable behaviors of the state variables are different from each other and asymptotic convergence behavior is not ideal for all the state variables. Therefore, based on the partial stability theorem, a new robust missile guidance law was developed and its effectiveness in finite time interception of maneuvering targets was analytically shown. Also, the proposed guidance law provided better performance in comparison with the sliding mode guidance law.

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